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THE EFFECT OF COMPRESSIBILITY ON TWO-DIMENSIONAL TUNNEL-WALL INTERFERENCE FOR A SYMMETRICAL AIRFOIL

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THE EFFECT OF COMPRESSIBILITY ON TWO-DIMENSIONAL TUNNEL-WALL INTERFERENCE FOR A SYMMETRICAL AIRFOIL

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SUMMARY

The effective change in the velocity of flow past a wing section, caused by the presence of wind-tunnel walls, is known for potential flow. This theory is extended by investigation of the two-dimensional compressible flow past a thin Rankine Oval. It is shown that for a symmetrical section at zero angle of attack the velocity increment due to the tunnel walls in the incompressible case must be multiplied by the factor $1/1-M^2$ to take account of compressibility effects. The Mach number, M, corresponds to conditions in the wind-tunnel test section with the model removed.

INTRODUCTION

Present day high-speed operation of wind tunnels renders it important to study the effects of compressibility on the tunnel-wall interference. Several studies of the two-dimensional inviscid compressible flow in an infinite stream over sections derivable from a circle have been made. Two recent papers have demonstrated a method for extending these studies to flow bounded by a channel. Ernest Lamla (reference 1) considered the incompressible potential flow past a circular cylinder in a channel as a first order approximation and used the Janzen-Rayleigh method to find the compressible-inviscid flow past the circular cylinder in a channel. Von Hantzsche and Wendt (reference 2) have extended Lamla's study to the special family of ellipses in a channel having a maximum incompressible velocity past their surface equal to twice the free stream velocity.

These special cases are far removed from the usual conditions of wind-tunnel experimentation. A Rankine Oval having a thickness ratio comparable to a symmetrical

airfoil section gives a reasonable representation of the effects of profile thickness on the tunnel-wall velocity corrections for an airfoil. A. Fage (reference 3) reports the solution, by Sir Richard Glazebrook, of the incompressible potential flow over a Rankine Oval in a channel. The present report takes this solution as a first order approximation and applies an analysis paralleling that of Lamla's to find the two-dimensional compressible inviscid flow past a thin Rankine Oval in a channel.

THEORY

In the Janzen-Rayleigh method (reference 1), the general stream function, and thus the velocity V at any point in the flow, is expressed as an infinite series function of the free stream Mach number M (corresponding to the free stream velocity V_o), as

$$\psi = \sum_{\eta=0}^{\infty} \psi_\eta M^{2\eta}$$

so that

$$\left(\frac{V}{V_o}\right)^2 = K = \sum_{\eta=0}^{\infty} K_\eta M^{2\eta}$$

where

$$K_o = \left(\frac{\partial\psi_o}{\partial x}\right)^2 + \left(\frac{\partial\psi_o}{\partial y}\right)^2$$

The potential flow stream function ψ_o satisfies Laplace's equation $\Delta^2\psi_o = 0$. Introducing the conditions of continuous and irrotational flow, leads to

$$\Delta^2\psi_1 = -\frac{1}{2}\left[\frac{\partial\psi_o}{\partial x}\frac{\partial K_o}{\partial x} + \frac{\partial\psi_o}{\partial y}\frac{\partial K_o}{\partial y}\right] \tag{1}$$

Therefore the second approximation to the general stream function

$$\psi = \psi_o + \psi_1 M^2$$

can be found from the first approximation $\psi = \psi_o$.

For adiabatic changes, to the order of M^2,

$$\frac{\rho}{\rho_s} = \left[1 - \frac{\gamma - 1}{2} K_o M^2\right]^{\frac{1}{\gamma-1}} \cong 1 - \frac{1}{2} K_o M^2$$

and

$$\frac{\rho_o}{\rho_s} = \left[1 - \frac{\gamma - 1}{2} M^2\right]^{\frac{1}{\gamma-1}} \cong 1 - \frac{1}{2} M^2$$

consequently

$$\frac{\rho_o}{\rho} \cong 1 + \frac{K_o - 1}{2} M^2$$

where

ρ fluid density at any point in the stream

ρ_o fluid density in the undisturbed stream

ρ_s fluid density at stagnation point

The x velocity component at any point in the flow is

$$\frac{V_x}{V_o} = \frac{\rho_o}{\rho} \frac{\partial\psi}{\partial y} = \frac{\partial\psi_o}{\partial y} + \left[\frac{\partial\psi_o}{\partial y} \frac{(K_o - 1)}{2} + \frac{\partial\psi_1}{\partial y}\right] M^2 \tag{2}$$

when terms having powers of M greater than the square are neglected.

ANALYSIS

The potential stream function for a Rankine Oval symmetrically placed in a channel (reference 3) is

$$\psi_o = y + A \tan^{-1}\left[\frac{\sinh r \sin m}{\cosh \lambda - \cosh r \cos m}\right]$$

where

$$A = \frac{-\,t}{2\tan^{-1}\left(\dfrac{\sinh r \sin\left(\frac{tr}{s}\right)}{1 - \cosh r \cos\left(\frac{tr}{s}\right)}\right)}$$

and all angles must be taken to lie between 0 and π.

t thickness of the Rankine Oval

s distance between source and sink producing Rankine Oval

V_0 velocity of undisturbed stream

$l = \frac{2\pi x}{H}$, $m = \frac{2\pi y}{H}$, $r = \frac{\pi s}{H}$

H breadth of channel

x coordinate measured along axis of the channel (major axis of oval) and having the origin midway between source and sink

y coordinate measured perpendicular to the axis of the channel and having the origin midway between the tunnel walls

Applying equation (1) gives

$$\Delta^2\psi_1 = -\frac{2\pi}{H}\sin m\left\{\frac{E^3}{D^2}\cosh l - \frac{2E^2}{D^2}(\cosh r\cosh l - \cos m)\right.$$

$$\left. - \frac{2E}{D^2}(\cosh l - \cosh r\cos m)\sinh^2 l + \frac{E}{D}\cosh l\right\}$$

where

$$E = \frac{2\pi A}{H}\sinh r$$

$$D = (\cosh l - \cosh r\cos m)^2 + (\sinh r\sin m)^2$$

To solve this partial differential equation by Lamla's method, it is necessary to obtain $\cos m$ as a finite

series in D, r, and l from the preceding equation. This imposes the limit $(\cosh^2 r - 1)$ must be less than 1. For the sake of simplicity set $\cosh r = 1 + e$, where e is a small quantity. Consider a solution of equation (1) having the form

$$\psi_1 = -\frac{H}{2\pi}\sum_{p=1}^{5}\frac{g_p\,(l)\,\sin m}{D^{p/2}}$$

Then obtain the functions $g_p(l)$ by equating those terms in the two expressions for $\Delta^2\psi_1$ which have the same powers of D. If terms in e^2 are neglected

$$\psi_1 = -\frac{H}{2\pi}\, I \sin m$$

where

$$I = \frac{d_0 E}{D^{\frac{1}{2}}} + \frac{d_1 E^2 \cosh l + (E/2)\, l \sinh l}{D} + \frac{(1 - \cosh 2l)}{D^{3/2}}$$

$$\left(\frac{E^2}{2} - \frac{ed_0 E}{2} + d_1 E^2\right) + \frac{ed_1 E^2}{D^{3/2}}\left(1 - \frac{\cosh 2l}{3}\right) + \frac{E^3}{12\, D^{3/2}}\,(1 - e)\cosh 2l$$

On the surface of the Rankine Oval $\psi_1 = o$, this provides the conditions necessary for evaluating the two constants d_0 and d_1. For convenience the coordinates for the ends of the Oval axes can be used

$$I = o \quad \text{when} \begin{cases} l = o, & m = \dfrac{\pi t}{H} \\ m = o, & l = \cosh^{-1}(-E + \cosh r) \end{cases}$$

This completely determines ψ_1 and it is now possible to calculate the velocity at any point in the flow from equation (2) (and the analogous equation for the y velocity component).

The characteristic point on the Rankine Oval may be taken to be the point $x = o$, $y = t/2$. For this point

$$\left(\frac{V_x}{V_o}\right)_{x=o} = \left\{1 - \frac{E}{D^{\frac{1}{2}}}\right\}_{x=o} + M^2\left[\left\{1 - \frac{E}{D^{\frac{1}{2}}}\right\}\left\{\frac{1}{2}\frac{E^2}{D} - \frac{E}{D^{\frac{1}{2}}}\right\} - \sin\frac{\pi t}{H}\left(\frac{\partial I}{\partial m}\right)\right]_{x=o} \tag{3}$$

Lanla's result for a circle is the special case obtained from equation (3) by taking $e = o$.

It can be shown that

$$\sin \frac{\pi t}{H} \left(\frac{\partial I}{\partial m}\right)_{x=o} = a\ (t/s)^4 \tag{4}$$

and

$$\left\{1 - \frac{E}{D^{\frac{1}{2}}}\right\} \left\{\frac{1}{2}\frac{E^2}{D} - \frac{E}{D^{\frac{1}{2}}}\right\}_{x=o} = b\ (t/s) \tag{5}$$

The most frequent applications have a thickness ratio, t/c, less than 0.3, where

$$\frac{t}{c} = \frac{t/s}{\left[1 + \dfrac{t/s}{\cot^{-1}(t/s)}\right]^{\frac{1}{2}}}$$

therefore (4) would be negligible in comparison with (5) since a and b are of the same order. In such cases it is unnecessary to evaluate Ψ_1 and

$$\left(\frac{V_x}{V_o}\right)_{x=o} = \left\{1 - \frac{E}{D^{\frac{1}{2}}}\right\}_{x=o} + M^2 \left[\left\{1 - \frac{E}{D^{\frac{1}{2}}}\right\} \left\{\frac{1}{2}\frac{E^2}{D} - \frac{E}{D^{\frac{1}{2}}}\right\}\right]_{x=o} \tag{6}$$

In reference 2 it is shown that for an ellipse having t/c less than 0.3, in a channel with breadth so chosen as to give at $x=o$ the incompressible velocity past the ellipse of twice the free stream velocity, $\left(\frac{V_x}{V_o}\right)_{x=o}$ is very nearly $2+3M^2$. For 0.3 thickness ratio the velocity at $x=o$ past a Rankine Oval is within 10 percent of that past an ellipse. In equation (6) set $M=o$, and take

$$\left(\frac{V_x}{V_o}\right)_{x=o,M=o} = \left\{1 - \frac{E}{D^{\frac{1}{2}}}\right\} = 2$$

or

$$\frac{E}{D^{\frac{1}{2}}} = -1$$

then

$$\left(\frac{V_x}{V_o}\right)_{x=o} = 2+3M^2$$

which gives a check on the validity of equation (6).

A further simplification in the application of equation (6) is

$$\left(\frac{E}{D^{\frac{1}{2}}}\right)_{x=o} = \frac{-2(t/s)\left[1+e/6(1+(t/s)^2)\right]}{(1+(t/s)^2)\left[\pi - \frac{2\ t/s}{1-(t/s)^2}\left(1-\frac{4}{3}\left(\frac{t}{s}\right)^2+\frac{e}{6}\left(1-\left(\frac{t}{s}\right)^2\right)\right)\right]} \tag{7}$$

Comparison of this value with table I, reference 3, shows that for $M=o$, t/s less than 0.4, and s/H less than 0.5, the values for $\left(\frac{V_x}{V_o}\right)_{x=o}$ check within 0.1 percent. If an accuracy of 0.5 percent is sufficient, equation (7) can be reduced to

$$\left(\frac{E}{D^{\frac{1}{2}}}\right)_{x=o} = \frac{-\frac{2t}{\pi c}}{1-\frac{2t}{\pi c}}\left[1+\frac{\left(\frac{\pi c}{H}\right)^2}{12}\left(\frac{1}{1-\frac{2t}{\pi c}}\right)\right] \tag{8}$$

Setting equation (8) in (6) it follows that at $x=o$ the velocity past the Rankine Oval is

$$\left(V_x\right)_{x=o} = \frac{V_o}{1-\frac{2t}{\pi c}}\left\{\left[1+\frac{\left(\frac{\pi c}{H}\right)^2}{12}\left(\frac{2t/\pi c}{1-2t/\pi c}\right)\right]\right.$$

$$\left.+M^2\left[\frac{2t/\pi c}{1-3t/\pi c}\left(1+\frac{\left(\frac{\pi c}{H}\right)^2}{12}\left(1+\frac{5t}{\pi c}\right)\right)\right]\right\} \tag{9}$$

within the limits t/c less than 0.30 and c/H less than 1/4. From equation (9) it is seen that the velocity past the Rankine Oval at $x=o$ in an infinite stream $\left(\frac{c}{H} = o\right)$ is a function of Mach number, therefore in calculating the tunnel velocity correction it is necessary to consider the Mach number correction (reference 1)

$$\left(V_x\right)_{x=o} = \frac{V_o + \Delta V}{1 - 2t/\pi c}\left[1 + (M + \Delta M)^2\left(\frac{2t/\pi c}{1 - 3t/\pi c}\right)\right] \qquad (10)$$

Writing equation (9) in the form of equation (10) gives as the tunnel velocity correction for thin bodies

$$\left(\frac{\Delta V}{V_o}\right)_{x=o} = [1 + M^2]\frac{\left(\frac{\pi c}{H}\right)^2}{12}\left(\frac{2t}{\pi c}\right) \qquad (11)$$

In reference 1, it is demonstrated that for a circle

$$\left(\frac{\Delta V}{V_o}\right)_{x=o} = [1 + \frac{11}{6}M^2]\frac{\left(\frac{\pi c}{H}\right)^2}{12}$$

This gives an indication of the magnitude of change in the multiplying constant of M^2 for very thick bodies.

In reference 3 it is shown that for c/H less than 0.5, the percentage tunnel velocity correction for all points on the body is nearly constant. If this limit is exceeded, the restriction of the tunnel walls causes a change in the effective shape of the body, because the velocity correction varies from point to point on the surface of the cylinder.

A comparison of equation (11) with incompressible flow tunnel velocity correction as given in reference 4 shows that

$$\left(\frac{\Delta V}{V_o}\right)_{compressible} = \left(1 + M^2\right)\left(\frac{\Delta V}{V_o}\right)_{incompressible} \qquad (12)$$

The present development neglects powers of M greater than the square.

The mathematical difficulties of extending the present analysis to higher Mach numbers are formidable. However, an investigation of the flow characteristics at Mach numbers approaching 1 suggests an extension of equation (12). To this end, consider the two-dimensional flow past a thin symmetrical airfoil in free air. The conditions of conservation of mass, and adiabatic compression lead to

$$\frac{l}{l_o} = \frac{M_o}{M}\left[\frac{1+\frac{\gamma-1}{2}M^2}{1+\frac{\gamma-1}{2}M_o^2}\right]^{\left(\frac{1}{2}+\frac{1}{\gamma-1}\right)} \tag{13}$$

where l_o and l are, respectively, the distances between the same two stream lines ahead of the airfoil and at the position of maximum velocity over the airfoil. The velocities at these positions correspond to the Mach numbers M_o and M. γ is the ratio of the specific heats.

The maximum velocity past the surface of a thin symmetrical section is of the same magnitude as the velocity ahead of the section. Therefore, for small Mach numbers, equation (13) indicates that the distances between streamlines at these two positions are inversely proportional to their local Mach numbers. This corresponds to a natural contraction of the streamlines.

As the Mach numbers approach 1, $\frac{l}{l_o}$ approaches 1. To demonstrate this, let

$$M_o = 1 - \epsilon \quad \text{and} \quad M = 1 - \delta,$$

where ϵ and δ are small quantities. Then, since $\gamma = 1.4$ for air, equation (13) can be written

$$\frac{l}{l_o} = \frac{1-\epsilon}{1-\delta}\left[\frac{1+0.2\,(1-2\delta)}{1+0.2\,(1-2\epsilon)}\right]^3 \cong \left(\frac{1-\epsilon}{1-\delta}\right)\left(\frac{1-\delta}{1-\epsilon}\right) = 1$$

The physical significance of this relation is that in the vicinity of the velocity of sound, the streamlines in passing from the undisturbed stream over the airfoil section would not contract in free air. The presence of

tunnel walls imposes a contraction, which alters the flow markedly. To the order of accuracy of this report, $1 + M^2$ is the series equivalent to $\frac{1}{1 - M^2}$ and the latter form expresses the large magnitude of the compressibility correction when M approaches 1. Therefore, it seems advisable to write equation (12)

$$\left(\frac{\Delta V}{V_o}\right)_{compressible} = \frac{1}{1 - M^2}\left(\frac{\Delta V}{V_o}\right)_{incompressible.}$$

This relation was derived for a thin Rankine Oval and indicates the order of correction for a thin symmetrical airfoil section at zero angle of attack in a two-dimensional wind tunnel.

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